<u>An introduction to SPACKlick's Information modelling solution to the MHP</u>

<u>*Intro*</u>

> *"Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?"*

I add the standard assumptions; the host must always open a door, it cannot be the chosen door, the host must always reveal a goat, the host must always offer the contestant the switch. I assume nothing about the strategy the host uses for deciding which door to open when the contestant chooses the car.

Non-Standard Notation

X{a,b,c…} is Object X is known to exist and is known to have properties a, b, c etc.Objects are named when first encountered but may resolve to be a second label for an existing object. Properties in ***red italics*** are unknowns and are represented by either the property name in question marks or an arbitrary label. Where an unknown is ***<u>underlined</u>***, the contestant knows they will not know it at the final choice.

$\mathbf{D}_i$ is the set of properties of the $\mathbf{Door}_i$ not including relative position, closed state, chosen state or what it conceals [I've listed those separately] accessible to the host. $:\sqsupset\mathbf{D}_i$ is that subset of $\mathbf{D}_i$ which is accessible from the contestants POV. I feel comfortable making the contestants knowledge a subset of the hosts because the host has at least the same, if not more, perceptual access to the doors as the contestant.

$\mathbf{G}_i$ is the set of properties of $\mathbf{Goat}_i$ not including Concealed state, Door location or chosen state [I've listed those separately]. As with $:\sqsupset\mathbf{D}_i$, $:\sqsupset\mathbf{G}_i$ is those properties of the goat available to the contestant when the goat is revealed.

{a,b,c} = {x,y,z} means the two sets contain the same elements, not necessarily in the same order.

For all of the below I am elaborating only the case where the car is behind the door on the left. It is trivial to show by symmetry that this applies wherever the car is.

<u>*Information (Host)*</u>

Initial

1. **Door$_1${On the left, Closed, Unchosen, Conceals Car, D$_1$}**
2. **Door$_2${In the Middle, Closed, Unchosen, Conceals Goat$_2$, D$_2$}**
3. **Door$_3${On the Right, Closed, Unchosen, Conceals Goat$_1$, D$_3$}**
4. **Car{Concealed, Behind Door$_1$, Unchosen}**
5. **Goat$_1${Concealed, Behind Door$_3$, Unchosen, G$_1$}**
6. **Goat$_2${Concealed, Behind Door$_2$, Unchosen, G$_2$}**
7. Contestant will choose a door [**Door$_z${*?position?*, Closed, Chosen, *?conceals?*, D$_z$}**]
8. I will open a door by the rules [**Door$_x${*not left*, Open, Unchosen, *Conceals Goat$_n$*, D$_x$}** where x ≠ 1] (based on any preferences for any and all traits of the doors and goats)
9. Revealing a goat [**Goat$_n${Revealed, Behind Door$_x$, Unchosen, G$_n$}**]
10. The contestant will then Choose between **Door$_1${On the left, Closed, *?chosen?*, Conceals Car, D$_1$}** to win the **Car** and **Door$_y${*?position?*, Closed, *?chosen?*, *Conceals Goat$_m$*, *D$_y$*}** ***Goat***

At the final choice assuming Door 1 picked first

1. **Door$_1${On the left, Closed, Chosen, Conceals Car, D$_1$}**
2. **Door$_2${In the Middle, Closed, Unchosen, Conceals Goat$_2$, D$_2$}**
3. **Door$_3${On the Right, Opened, Unchosen, Conceals Goat$_1$, D$_3$}**
4. **Car{Concealed, Behind Door$_1$, Unchosen}**
5. **Goat$_1${Revealed, Behind Door$_3$, Unchosen, G$_1$}**
6. **Goat$_2${Concealed, Behind Door$_2$, Unchosen, G$_2$}**

7. The contestant will now stick with **Door$_1${On the left, Closed, Chosen, Conceals Car, D_1}** to win the **Car** or switch to **Door$_2${In the Middle, Closed, Unchosen, Conceals Goat$_2$, D_2}** to win **Goat$_2$**

   x=3, y=2, z=1, n=1, m=2

At the final choice assuming Door2 picked first

1. **Door$_1${On the left, Closed, Unchosen, Conceals Car, D_1}**
2. **Door$_2${In the Middle, Closed, Chosen, Conceals Goat$_2$, D_2}**
3. **Door$_3${On the Right, Opened, Unchosen, Conceals Goat$_1$, D_3}**
4. **Car{Concealed, Behind Door$_1$, Unchosen}**
5. **Goat$_1${Revealed, Behind Door$_3$, Unchosen, G_1}**
6. **Goat$_2${Concealed, Behind Door$_2$, Chosen, G_2}**
7. The contestant will now stick with **Door$_2${In the Middle, Closed, Chosen, Conceals Goat$_2$, D_2}** to win **Goat$_2$** or switch to **Door$_1${On the left, Closed, Unchosen, Conceals Car, D_1}** to win the **Car**

Information (Contestant)

Initial

1. **Door$_1${On the left, Closed, Unchosen, *?conceals?*, :⊐D_1}**
2. **Door$_2${In the Middle, Closed, Unchosen, *?conceals?*, :⊐D_2}**
3. **Door$_3${On the Right, Closed, Unchosen, *?conceals?*, :⊐D_3}**
4. **Car{Concealed, *behind Door$_a$*, Unchosen}**
5. **Goat$_1${Concealed, *behind Door$_b$*, Unchosen, *:⊐G_1*}**
6. **Goat$_2${Concealed, *behind Door$_c$*, Unchosen, *:⊐G_2*}**
7. **{a,b,c} = {1,2,3}**
8. I will choose a door [**Door$_z${*?position?*, Closed, Chosen, *?conceals?*, :⊐D_z}**]
9. The Host will open a door **[Door$_x${*?position?*, Open, Unchosen, conceals Goat$_n$, :⊐D_x}]**
10. revealing a goat[**Goat$_n${Revealed, *behind Door$_x$*, Unchosen, *:⊐G_n*}]**
11. **$x \neq a$**
12. I can then stick with **Door$_z${*?position?*, Closed, Chosen, *?conceals?*, :⊐D_z}** or switch to **Door$_y${*?position?*, Closed, Unchosen, *?conceals?*, :⊐D_y}** to win the concealed prize in it.
13. **{x,y,z} = {1,2,3}**

At the final choice Assuming Door 1 picked first

1. **Door$_1${On the left, Closed, Chosen, *?conceals?*, :⊐D_1}**
2. **Door$_2${In the Middle, Closed, Unchosen, *?conceals?*, :⊐D_2}**
3. **Door$_3${On the Right, Open, Unchosen, conceals Goat$_n$, :⊐D_3}**
4. **Car{Concealed, *behind Door$_a$*, Unchosen}**
5. **Goat$_1${Concealed, *behind Door$_b$*, Unchosen, *:⊐G_1*}**
6. **Goat$_2${Concealed, *behind Door$_c$*, Unchosen, *:⊐G_2*}**
7. **{a,b,c} = {1,2,3}**
8. **$3 \neq a$**
9. **Goat$_n${Revealed, behind Door$_3$, Unchosen, :⊐G_n}**
10. I can now stick with **Door$_1${On the left, Closed, Chosen, *?conceals?*, :⊐D_1}** or switch to **Door$_2${In the Middle, Closed, Unchosen, *?conceals?*, :⊐D_2}** to win the concealed prize in it.

At the final choice assuming Door2 picked first

1. **Door$_1${On the left, Closed, Unchosen, *?conceals?*, :⊐D_1}**
2. **Door$_2${In the Middle, Closed, Chosen, *?conceals?*, :⊐D_2}**
3. **Door$_3${On the Right, Open, Unchosen, conceals Goat$_n$, :⊐D_3}**
4. **Car{Concealed, *behind Door$_a$*, Unchosen}**
5. **Goat$_1${Concealed, *behind Door$_b$*, Unchosen, *:⊐G_1*}**
6. **Goat$_2${Concealed, *behind Door$_c$*, Unchosen, *:⊐G_2*}**
7. **{a,b,c} = {1,2,3}**
8. **Goat$_n${Revealed, behind Door$_3$, Unchosen, :⊐G_n}**
9. **$3 \neq a$**
10. I can now stick with **Door$_2${In the Middle, Closed, Chosen, *?conceals?*, :⊐D_2}** or switch to **Door$_1${On The Left, Closed, Unchosen, *?conceals?*, :⊐D_1}** to win the concealed prize in it.

Conditional Summary

All probabilities are "*given the initial information*" so P(x) means P(x|I). Conditioning on X is only meaningful if X is not contained in I. The contestant, in the time where he chooses door 1, has learned the following; that was not in I

- The value of z [Which door the contestant first chose]
- The value of x [Which door the Host Opened]
- The value of y [By elimination, the Other door]
- $Goat_n$ has specific traits including ≕G_n

These are the only things it is meaningful to condition on. All three doors are, in I, identical bar position. As position plays no role in any other information it is trivial to show which door is which does not affect the probability from the contestants POV. Although it may do for the host.

So the real question is what is the meaning of P(C=2 | $Goat_n${≕G_n}). The probability the car is behind door 2, given that the goat has some trait(s), say "*is black*". Nothing in I is dependent on ≕G_n as is demonstrated by considering any specific value for ≕G_n and observing that it only appears in the same line in each event with no other ≕G_x to compare it to.

To put this another way, at the Initial State, the contestant knows they will observe **$Goat_n${≕G_n}**, They also know that it will be the only **≕G_x** they observe. So they know that the infinite possible sets **{≕G_n}** divide the probability tree into infinite identical branches the contents of which are independent of the value **≕G_n**.

From this I conclude that the only possible conditional probability to consider is;

P(y=a|z= {1,2,3}, x={1,2,3}, y={1,2,3}, ≕G_n={some traits})

and that from the contestants POV ;

P(y=a|z= {1,2,3}, x={1,2,3}, y={1,2,3}, ≕G_n={some traits}) ≡ P(y=a)

for any valid combination of values. To calculate P(y=a) from the information above.

1. {1,2,3} = {x,y,z}
2. {1,2,3} = {a,b,c}
3. From 1 and 2 {x,y,z}={a,b,c}
4. No distribution is specified for z across {a,b,c} or {1,2,3}
5. Under all distributions, z is, on average, equiprobable across {a,b,c}
6. From 3 If z=a then y≠a.
7. x≠a
8. From 7 If z=b then x=c
9. From 7 z=c then x=b,
10. From 3 and 8 If z=b then y=a
11. From 3 and 9 if z=c then y=a
12. From 5, 6, 10 and 11 P(y=a)=2/3

To show this is identical to any of the givens above consider P(y=a|z=1, x=3, ≕G_n=white)

1. {1,2,3} = {x,y,z} (also x=3, y=2, z=1)
2. {1,2,3} = {a,b,c}
3. From 1 and 2 {x,y,z}={a,b,c}
4. No distribution is specified for z across {a,b,c}
5. Under all distributions, z is, on average, equiprobable across {a,b,c}
6. From 3 If z=a then y≠a.
7. x≠a
8. From 7 If z=b then x=c
9. From 7 z=c then x=b,
10. From 3 and 8 If z=b then y=a
11. From 3 and 9 if z=c then y=a
12. From 5, 6, 10 and 11 P(y=a)=2/3

Notice this is almost identical in its reasoning to the unconditioned case as none of the conditions affect the reasoning.